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Probing The Structure of Space-Time with Cosmic Rays

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Abstract

The study of the interactions of Cosmic Rays (CR's) with universal diffuse background radiation can provide very stringent tests of the validity of Special Relativity. The interactions we consider are the ones characterized by well defined energy thresholds whose energy position can be predicted on the basis of special relativity. We argue that the experimental confirmation of the existence of these thresholds can in principle put very stringent limits on the scale where special relativity and/or continuity of space-time may possibly break down.

I. INTRODUCTION

The study of interactions of high energy Cosmic Rays (nucleons and gamma rays) can provide severe tests of the validity of Special Relativity (SR). Indeed, SR is at the very base of our theories for the description of the Universe, and its validity is generally not questioned. However, several attempts have been made to put under scrutiny the postulates that SR

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relies upon, like the constancy of the velocity of light, and these studies have contributed accurate limits on possible violations (see e.g. [1] and references therein).

Although it is very well proven that SR and the underlying Lorentz invariance (LI) provide a suitable framework for our low energy effective theories, it is not clear whether more ambitious theories, aiming to a global description of our World, including gravity, are or need to be Lorentz invariant. Theoretically, the need for a unified theory of gravity has led to several models, some of which automatically imply the breaking of LI. This is qualitatively understandable since the very concept of continuous space-time is likely to be profoundly changed by quantum gravitational effects [2]. In this prospective we think it is worth keeping an open mind with respect to the validity of what we define as fundamental theories, and always put them under experimental scrutiny. This attitude appears even more justified at the present time: observationally, high energy astrophysics is providing a range of opportunities to probe energies (*i.e.* Lorentz factors, or speeds) much larger than the ones ever obtained, or obtainable in the future, in accelerator experiments. In particular we concentrate our attention on the interactions of cosmic gamma rays and nucleons with some type of universal photon background, *i.e.* the cosmic microwave (CMB), the far infrared (FIRB), and the radio background. These are the interactions responsible for gamma ray absorption from distant sources (through production of e^+e^-) [3–5], that should appear as a cutoff in the gamma ray spectrum, and of the well known Greisen-Zatsepin-Kuzmin (GZK) cutoff [6,7] due to the photopion production in collisions of ultra high energy cosmic rays (UHECRs) off the CMB photons. All these processes have the same general structure: in a Lorentz invariant picture, these reactions would be examples of very low energy processes in the center of momentum, that appear boosted to very large Lorentz factors in the laboratory frame. Testing the presence of the thresholds for these processes is therefore a test of SR up to the large Lorentz factors in the boost.

Our approach is entirely phenomenological, and reasonably model independent. We do not propose, if not as examples, any specific model. We show that when a very general

form of LI violation [1] is explicitly allowed in the dispersion relation between energy and momentum of particles, the threshold momenta for some of these reactions may drastically change or even become unphysical (*i.e.* the process does not occur), unless the LI violation is introduced at a length scale much smaller than the Planck scale. A similar approach, although more specific, was proposed in [8] (see Sect. 3).

In our opinion, the (lack of) knowledge of the sources of the CR's under consideration does not allow us to support the idea [1,9] that the present experimental situation gives evidence for violation of LI or of continuity of space-time. Rather we stress that a verification of the existence of the quoted thresholds would entail a lower limit on the mass scale of effective LI violations.

The paper is planned as follows: in section 2 we discuss our parametrization of the LI violations, in section 3 we apply our calculations to the case of pair production and photopion production in high energy cosmic ray interactions. In section 4 we provide explicit examples of theoretical models that predict a broken LI. We conclude in section 5.

II. BREAKING OF LI AND THE CALCULATION OF THE PARTICLE PRODUCTION THRESHOLDS

Lorentz invariance implies that the modulus of any four-vector is unchanged when changing reference frame; for instance for the four-momentum of a particle we have (we always

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[1]Clearly, breaking LI can have several implications, as for instance the existence of a preferred reference frame, which is the one in which all the calculations need to be carried out, since breaking LI also invalidates the transformations that allow us to change reference frame. This reference frame could be identified with the one comoving with the expansion of the universe (no peculiar motion), in which the microwave background is completely isotropic. In fact, there is only one frame with this property, being all other frames experiencing the dipole anisotropy, and therefore distinguishable.

put $c = \hbar = 1$)

$$P_\mu P^\mu = E^2 - p^2 = const = m^2$$

m being the invariant particle mass and $p = |\vec{p}|$.

Violations of Lorentz invariance will therefore affect in general the *dispersion relation* above.

We write a modified dispersion relation obeying the following postulates:

1. Violations are a *high energy* effect, *i.e.* they vanish at small momenta.

2. Violations are universal, *i.e.* do not depend on the particle type, if not (possibly) through the particle mass.

3. Rotation invariance remains exact.

Clearly requirements 2 and 3 may be relaxed and in fact there are examples in this sense [1,10].

In a general way we can write the modified dispersion relation as follows:

$$E^2 - p^2 - m^2 = p^2 f(\frac{p}{M}) + m^2 g(\frac{p}{M}) \tag{1}$$

where the mass M parametrizes the violation of Lorentz invariance (or an essential discreteness of space-time, as for instance suggested by some models of quantum gravity (e.g. [2] and [11,12])).

Even in the framework defined above this is not the most general violation term. However, in the regime we are interested in, $m \ll p \ll M$, the left hand side of eq. (1) is small compared to p and E so that the other possible terms we can write (containing for instance E) differ from those in eq. (1) by higher order corrections.

Since $p/M \ll 1$ the functions f and g can be Taylor-expanded to give:

$$E^2 - p^2 - m^2 = p^2(f(0) + f'(0)\frac{p}{M} + f''(0)\frac{p^2}{M^2} + ...)+$$

$$m^2(g(0) + g'(0)\frac{p}{M} + g''(0)\frac{p^2}{M^2} + ...) \tag{2}$$

In the limit $M \to \infty$ one must recover the Lorentz invariant dispersion relation, so $f(0) = g(0) = 0$; moreover the linear term might be absent, as we will see later; if it is present, the quadratic term is negligible at the momenta we consider. The coefficients in front of the first and second derivatives are left unconstrained. However, if $|f'(0)|, |g'(0)| \gg 1$ or $|f''(0)|, |g''(0)| \gg 1$, then the functions f and g would be strongly varying and possibly oscillating, which would be unphysical. Moreover, in cases where the calculations of the LI breaking can be carried out explicitly (see section 4), these coefficients turn out to be of order unity (the same motivation justifies the assumption that these coefficients are not much smaller than unity). Therefore we decided to adopt here a phenomenological approach and assume that possible small deviations from unity are embedded in the mass scale M. The validity of this approach will be checked *a posteriori*.

We are thus led to the following classification of Lorentz non-invariant dispersion relations:

$$I_{\pm} : \qquad E^2 - p^2 \approx m^2 \pm \frac{p^3}{M} \quad (\pm\frac{m^2 p}{M}) \tag{3}$$

$$II_{\pm} : \qquad E^2 - p^2 \approx m^2 \pm \frac{p^4}{M^2} \quad (\pm\frac{m^2 p^2}{M^2}) \tag{4}$$

where I (II) stands for first (second) order modification and the terms in parenthesis come from the expansion of g.

We can define a critical momentum p_c where the correction, for massive particles, equals m^2, which is the momentum for which we expect that deviations from normal relativistic kinematics become relevant. We have $p_c \approx 2 \times 10^{15}$ eV ($\approx 10^{13}$ eV) for protons (electrons) in the case $I_{\pm}$ and $p_c \approx 3 \times 10^{18}$ eV ($\approx 10^{17}$ eV) for protons (electrons) in the case $II_{\pm}$. In the cases in parenthesis in eqs. (3) and (4) we have always $p_c \approx M$, so these modifications do not lead to observable consequences at the energies we are interested in; we will therefore put $g(\frac{p}{M}) = 0$ in the following. We will come back to this point when we will describe specific examples of Lorentz violating theories [8]. It is clear that the values of p_c given above are calculated in a specific frame; in fact, generally, violations of LI imply

the existence of a privileged frame. We choose this frame to be the one comoving with the expansion of the universe (we name it "universal frame"), and we argue that this choice is in fact not arbitrary: this is the only possible frame where the microwave background is isotropic (the same holds for the other backgrounds provided the sources are homogeneously and isotropically distributed). Moreover, neglecting the proper motion of the Earth, this is the reference frame in which we live and measure the thresholds for physical processes. On a more practical ground, it is worth noticing that giving up LI, the possibility to perform a transformation to a different frame is precluded, although in principle it is possible to write modified Lorentz transformations including the LI violations at the perturbative level, as done before for the dispersion relation. In this case, however, there is much more arbitrariness than in modifying the dispersion relation and we do not pursue this approach in this paper. It is worth noting however that the Lorentz Transformations are derived in a LI theory by the requirement of the invariance of a fundamental interval, so in a sense a modification of the dispersion relation is more fundamental.

There is a further, important point to discuss before computing particle production thresholds. In fact, when one gives up relativistic invariance, energy-momentum conservation is not anymore guaranteed, and relativistic quantum field theory may fail, so there is no guidance in deriving cross sections. However, our point of view here is to derive the consequences of experimental *verification* of the existence of the particle thresholds, so we will *assume* in the following exact energy-momentum conservation *and* relativistic dynamics in the preferential frame. Further discussion on this fundamental issue will be postponed to section 3 and conclusions.

A. e^+e^- production in photon-photon interactions

The energy of the background photons is taken as $\omega = |k|$ (where k is the photon momentum) since at the typical background momenta (FIRB: $\omega \approx 0.01$ eV, CMB: $\approx 6 \times 10^{-4}$ eV, and radio: $\approx 4 \times 10^{-9}$ eV for the peak of the corresponding radiance distributions) the

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corrections are entirely negligible. We compute the threshold, assuming that the CR particle and the background photon collide head-on, and the final particles are collinear. This is indeed not an arbitrary configuration, but the one that provides the minimum energy for which the process can occur in the universal frame. Rotational invariance then implies that in this reaction the momenta of final particles are equal, and that particles move in the direction of the primary, so that the problem is one-dimensional; the energy of the background photon is assumed to be equal to the average for the photon background in consideration.

Let then (E, p) be the energy and (modulus of) the momentum of the incident photon; after some trivial algebra we get, at the leading order in the correction, the following general equations for the threshold:

$$I_\pm : \quad \pm \alpha_I x^3 + x - 1 = 0, \tag{5}$$

$$II_\pm : \quad \pm \alpha_{II} x^4 + x - 1 = 0, \tag{6}$$

where $x = p_{th}/p_0$, $p_0 = m^2/\omega$ is the threshold for $M \to \infty$ (i.e. the usual threshold, $\approx 3 \times 10^{13}$ eV for the FIRB as background, $\approx 5 \times 10^{14}$ eV for the CMB, and finally $\approx 6 \times 10^{19}$ for the radio background), m the electron mass, and:

$$\alpha_I = \frac{p_0^3}{8 m^2 M}; \qquad \alpha_{II} = \frac{3 p_0^4}{16 m^2 M^2}. \tag{7}$$

The modified thresholds are the positive real solutions of the equations above, and in particular, if more than one positive solution is present, the one which goes to $x = 1$ as $M \to \infty$. In table I we report the solutions (if any) of the eqs. (5) and (6) in the case $M \approx M_P$, for the infrared, microwave, and radio background.

The general feature of these solutions is that a *positive* modification of the dispersion relation tends to move the thresholds towards lower momentum values; *negative* modifications tend to lead to *complex* solutions, meaning that the kinematics of the process becomes forbidden, *i.e.* the threshold disappears. Case II is less predictive, as it is obvious being

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the modification of second order in the (small) quantity p/M. Clearly a more detailed calculation, accounting for the integration over different scattering angles and over the whole spectrum of the photon background, could give slightly different results, not changing however our basic findings.

We now discuss what are the chances to use astrophysical observations in order to limit the possibility of non Lorentz invariant theories. In doing this it is important to keep in mind the quite different observational situation existing for the CMB versus the IR and Radio backgrounds: while the former is determined with very high accuracy, the latter are very poorly known and difficult to access through observational investigation, mainly due to the emission and/or absorption processes in our Galaxy.

Observationally, the infrared gamma-ray cutoff seems to be the one more easily approachable. Indeed signs of a cutoff in the TeV spectrum of a few blazars seem to be already present ([13–17]) and helped to impose some constraints on the extragalactic far-infrared background (FIRB). Unfortunately, as it is clear from table I, the case of interaction with the FIRB is not very predictive, in the sense that the threshold is not appreciably changed with respect to the case of a Lorentz-invariant theory, for most of the possible parametrizations of LI breaking (using $M = M_P$). The case I_- is an exception: in this case there is no acceptable solution of the equations that define the threshold, which means that the process is not possible at all. In other words, in this case no cutoff should be seen in astrophysical observations because photons can come unattenuated from all distances. Viceversa, if a cutoff is observed, then a lower limit on M can be imposed.

The case of the scattering off the CMB radiation is more useful: in the scenario I_+ the threshold is moved to smaller values. For $M = M_P$, photons with energy as small as ~ 25 TeV from a distant source should be absorbed. In order for the threshold to be unchanged, the scale M must exceed $\sim 800 M_P$. In the scenario I_- the process becomes not allowed, while in the other cases the threshold remains unchanged.

The case of scattering off the radio background is the most interesting, predicting either no threshold (cases I_- and II_-) or thresholds which are much smaller that the canonical

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ones. This could be of great importance for top-down models of UHECRs, where gamma rays are predicted to have an important role in the composition (e.g. see [18]). In particular the proton/gamma ratio is determined by the interaction of the gamma rays with the radio background at frequencies smaller than a few MHz. Unfortunately, as mentioned above, the radio background at these frequencies is extremely uncertain, and not accessible to any direct measurement, due to the strong free-free absorption in the disc and halo of our own Galaxy. Assuming however that in the future it will be possible to isolate the effect of pair production on different photon backgrounds and confirm the energy threshold within, say, a factor 2, then a lower limit on the scale M can be imposed. In table II we report the limits on M. Note that all these limits would be much more stringent than the few limits on M already obtained (see for instance [19]). Exceptions to this statement come from the cases $II_{\pm}$ for the IR and Radio backgrounds, where the limits on M are appreciably smaller that M_P, leaving more room for non Lorentz invariant theories.

B. Pion photoproduction in UHECR interactions: the GZK cutoff

This case is slightly more involved since the masses of the final particles are different, so that even for a rotation invariant modification, final momenta are not in the same ratio as the masses. However we checked that assuming the final momenta as in the Lorentz invariant theory, we introduce only higher order corrections. Using this prescription we obtain the following two equations for the threshold, with the same symbols used in the previous section:

$$I_{\pm} : \quad \pm\alpha_I x^3 + x - 1 = 0 \tag{8}$$

$$II_{\pm} : \quad \pm\alpha_{II} x^4 + x - 1 = 0 \tag{9}$$

where $x = \frac{p_{th}}{p_0}$ and $p_0 = \frac{m_\pi^2 + 2m_\pi m_p}{4\omega}$ is the conventional threshold (for $M \to \infty$), m_p is the proton (neutron) mass, and m_π is the pion mass. The coefficients $\alpha_{I,II}$ are defined as follows:

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$$\alpha_I = \frac{2p_0^3}{(m_\pi^2 + 2m_\pi m_p)M} \frac{m_\pi m_p}{(m_\pi + m_p)^2}; \tag{10}$$

$$\alpha_{II} = \frac{3p_0^4}{(m_\pi^2 + 2m_\pi m_p)M^2} \frac{m_\pi m_p}{(m_\pi + m_p)^2}.$$

Fixing $M = M_P$ we have the solutions reported in table III.

The general trend of the solutions is the same as in the case of pair production. Here however we can easily see that the consequences are even more evident: in all cases either the threshold disappears and the process becomes forbidden (cases I_- and II_-), or the threshold is appreciably lowered (cases I_+ and II_+). For I_- or II_- parametrizations, the GZK cutoff is completely washed out and particles (nucleons) should be able to reach us from any distance. In the other cases the threshold falls in a region that is easy to discard even on the basis of the present data, unless the scale of breaking of LI is $\sim 3 \times 10^{13}$ (~ 500) times larger than the Planck mass for the case I_+ (II_+). A complete list of the lower limits on M is reported in table III, from which it is clear that UHECR experiments soon to be available will provide a powerful tool to explore the very-small-scale structure of space-time.

III. SOME THEORETICAL EXAMPLES

As mentioned in section 2, our approach is based on the assumption that energy-momentum conservation is not affected by the violation of LI. In this section we provide a few examples of models that appear in some attempts to quantize gravity: we discuss energy-momentum conservation and show that dispersion relations of type $I_\pm$ and $II_\pm$ are actually obtained. We also discuss an alternative way of breaking LI, proposed by Kirzhnitz and Chechin in 1971 [8], that starts from a different point of view.

A. Quantum deformed Poincarè groups

Let us start from a model that leads to the II dispersion relation. In the context of theories that aim to quantize gravity, it is often the case that explicit breaking of LI is

found. For instance, applying quantum deformations to the Poincarè group one obtains a group with the so-called k-deformed Poincarè algebra [20].

The way to quantum-deform the Poincarè group is not unique, giving rise to several different k-deformed Poincarè algebras. The dispersion relation $II_\pm$ may be regarded as the eigenvalue of the Casimir operator of the following k-deformed Poincarè algebras [21,22]

- II_+ $\quad [N_i, P_j] = i\delta_{i,j} M \sin(\frac{P_0}{M}) \quad i,j = 1,2,3$

- II_- $\quad [N_i, P_j] = i\delta_{i,j} M \sinh(\frac{P_0}{M}) \quad i,j = 1,2,3$

leading to

- II_+ $\quad m^2 = M^2 \sin^2(\frac{E}{M}) - p^2 \sim E^2 - p^2 - \frac{E^4}{12M^2}$

- II_- $\quad m^2 = M^2 \sinh^2(\frac{E}{M}) - p^2 \sim E^2 - p^2 + \frac{E^4}{3M^2}$

where N are boosts and P_μ the generators of space-time translations; in the relations above we have reported only the sector of the algebra interested by the k-deformation, the rest of the algebra remaining unchanged. It is evident that in the limit $M \to \infty$ we recover the usual Poincarè algebra $[N_i, P_j] = i\delta_{ij} P_0$.

Assuming the possibility of constructing a *local* theory which is symmetric under the k-deformed Poincarè group [23], the conservation of energy and momentum follows, at the algebra level, from the commutation relation of the components P_μ of the generator of space-time translations. In both the cases quoted above we have the standard commutation relation $[P_\mu, P_\nu] = 0$ so that, in the case $II_\pm$, the energy-momentum would be conserved, assuming locality. It is not clear however if there are specific models in which this happens.

B. Quantum space-time fluctuations

A model that leads to the I dispersion relation also follows from quantum gravity. The basic idea of this model is that quantum fluctuations of gravity cause, at the scale of the Planck mass, the vacuum to behave like a stochastic medium (space-time foam) [2,11].

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This behaviour of the vacuum is described through an interaction between a closed-string state propagating in a $(D+1)$-dimensional space-time and a D(irichlet) particle embedded in this space-time. This interaction induces a recoil of the D particle which perturbs the surrounding space-time. During the recoil process the energy-momentum is conserved [11]. The distorsion of the space-time may be expressed through a modification of the usual flat metric with an energy-dependent non-diagonal term [12]. In the two cases $I_{\pm}$ one has:

- I_+: $g_{\mu\nu} \to g_{\mu\nu} + h_{0i}$ $h_{0i} = U_i$ $\vec{U} = -\frac{E}{M}\hat{p}$

- I_-: $g_{\mu\nu} \to g_{\mu\nu} + h_{0i}$ $h_{0i} = U_i$ $\vec{U} = \frac{E}{M}\hat{p}$

where E is the energy of the closed string excitations and $\hat{p}$ the versor of its momentum, M is the mass scale of the Lorentz invariance violation. The sign of the non-diagonal term depends on the kinematics of the D-brane recoil process. Also in this case it is evident that one recovers the usual flat metric in the limit $M \to \infty$.

The dispersion relations derived for this model can be written as follows (see also [24]):

- I_+ $m^2 = P^\mu g_{\mu\nu} P^\nu + P^\mu h_{\mu\nu} P^\nu = E^2 - p^2 - \frac{E^2}{M}p$

- I_- $m^2 = P^\mu g_{\mu\nu} P^\nu + P^\mu h_{\mu\nu} P^\nu = E^2 - p^2 + \frac{E^2}{M}p$.

At low energy $(E, p << M)$, point-like particles are represented by low energy excitations of the string, while the D-brane effect is embedded in the modification of the metric [11]. So the energy-momentum conservation in the recoil process ensures the energy-momentum conservation also at low energy.

C. Finslerian Spaces

There is a model of LI breaking, proposed in 1971 by Kirzhnits and Chechin [8], that does not follow from any Quantum Relativistic theory. The basic idea of this model is to replace Poincarè with Conformal invariance, and consequently the pseudo-Euclidean space-time with a Finslerian space [25].

In [8] the authors construct a theory which differs from the conventional one at sufficiently large values of the Lorentz factor, and which satisfies all the requirements of the conventional relativity. In other words, in this model the speed of light remains a fundamental constant and the principle of relativity is satisfied, i.e. the laws of motion are the same in all inertial frames. Moreover, this model, like the ones discussed above, is characterized by complete isotropy of space-time. We mention in passing that, in the context of a finslerian modification of space-time, there are some models [10] that imply a partial or total breaking of local spatial isotropy.

In the model proposed in [8] one arrives at the dispersion relation

$$m^2 = [1 + \bar{g}(\alpha \frac{p^2}{E^2})](E^2 - p^2), \tag{11}$$

where m is the particle mass, E and p are the energy and the momentum, and $\bar{g}$ is a homogeneous function of the dimensionless parameter α.

The difference between the model in [8] and the conventional (Lorentz invariant) theory is determined by the parameter α that, in terms of the mass scale M introduced before, may be expressed as

$$\alpha = \frac{m^2}{M^2} \tag{12}$$

were m is the particle mass. One recovers the usual theory in the limit $M \to \infty$ (like in the models cited above), but also in the limit $m \to 0$, so that the behaviour of massless particles is not affected, and the speed of light is a constant in any reference frame.

Being this model of violation of LI dependent on the particle mass, it is clear that it is milder because, compared to the models quoted above, introduces a suppression factor in the breaking term of the order of $\frac{m}{p}$. These modifications lead (with an appropriate choice of the function $\bar{g}$) to the terms in parenthesis in eqs. (3) and (4).

IV. CONCLUSIONS

Special Relativity is at the base of our understanding of the physical world. However any physical concept should always be put under stringent experimental verification. This is particularly true in the case of SR, specially in connection with the quest for a fundamental theory of Nature, which includes gravity. Any such theory will imply a full knowledge of what the vacuum really is at the level where quantum fluctuations build the space-time, possibly modelling it with a non-trivial geometry. The first attempts to reach this goal seem to suggest that Lorentz invariance, one of the building blocks of our current low energy theories, could be broken at extremely high energy.

Lacking a true theory of quantum gravity, the most we can do at present is to adopt a phenomenological approach and ask ourselves whether there is any probe that can be used to check the validity of SR at very high energy. This is precisely the approach adopted in this paper, where high energy cosmic rays (nucleons and gamma rays) have been used as probes. We found that the thresholds for pair production and photopion production off some universal photon backgrounds, which are experimentally accessible or will be in the next generation of cosmic ray experiments, are often profoundly affected by the possibility of breaking LI at supra-Planck scales. As a consequence, any quantum gravity theory in which LI is a casualty must face the cosmic ray bound in order to be viable.

Our calculations were based on a perturbative but quite general modification of the dispersion relation between energy and momentum of a particle. This modification affects in a fundamental way the calculation of the thresholds for pair production and photopion production, making these processes forbidden in some cases, or lowering the thresholds to questionable values in others. Building on this approach we proposed a procedure to obtain very strong constraints on the energy (or length) scale at which a possible LI violation could occur. This procedure relies upon the possibility that astrophysical observations will be able in the near future to find evidence for these processes, through cutoffs in the TeV spectra of distant blazars or/and through the discovery of the GZK cutoff in the spectrum of UHECRs

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or/and through studies of the composition of UHECRs. With a few exceptions, these limits are all higher than the Planck scale, which is an indication that breaking LI is not necessarily a safe ingredient of grand-unification theories that require it.

We stress that our approach is purely kinematical and no dynamical effect is considered. Moreover, we assume perfect energy-momentum conservation in order to compute the thresholds. While these are clearly important issues for any calculation aiming to *predict* values for specific particle production thresholds, and consequently absorption cutoffs in CR spectra, our approach is rather the opposite: we want to discuss the consequences of a possible experimental verification of the presence of particle production thresholds. If a given absorption threshold is experimentally detected, discarding the possibility of miraculous compensations between relativity violations, non conservation of energy-momentum and non-relativistically invariant dynamics, we are forced to conclude that possible violations of LI are smaller than the sensitivity of the experiment, and, to make this statement quantitative, we *choose* to use a purely kinematical, energy-momentum conserving parametrization.

Although a very tempting possibility, we do not believe that the current experimental situation allows us to draw definitive statements : recent observations of Markarian 501 might suggest the presence of a cutoff in the TeV region, but the unaffected spectrum is not known well enough to exclude that the observed effect is the artifact of a cutoff in the production spectrum. Hopefully the situation will improve with the next generation gamma ray detectors, complemented by neutrino and X-ray detectors that could clarify the origin of the gamma ray emission. As far as the photon absorption on the CMB radiation is concerned, the experimental situation is even more uncertain, due to the higher energy of the γ-rays involved (although there might be suggestion of attenuation due to CMB, see e.g. [26,27]).

Pair production on the universal radio background becomes relevant for ultra-high-energy gamma rays, usually produced in top-down models of UHECRs [18]. The presence or absence of a threshold for this process strongly affects our predictions of the fluxes at the Earth: although very uncertain, the radio background should allow typical gamma ray pathlengths

(in a Lorentz invariant world) of the order of $2 - 20$ Mpc [28]. If a violation of LI made the pair production kinematically forbidden, gamma rays could reach us from any distance and contribute an enormous flux of particles above the GZK cutoff.

As for UHECRs, although the (small) number of events [29,30] above $\sim 5 \times 10^{19}$ eV seems already incompatible with the presence of the GZK cutoff, the lack of knowledge of the sources does not allow any firm statement on the propagation of primaries in the Universe. The situation will improve dramatically in the near future, as new experiments like HiRes [31] and Auger [32] will collect data and reliable measurements on observables different from the energy spectrum (anisotropy, clustering) will be available with reasonable statistics.

Our conclusions can be summarized in the following points: 1) it is a 40-years old idea that the incompleteness of the present theories could rely upon our ignorance of the vacuum at very high energies. When we will have that knowledge we will probably have a quantum theory of gravity; 2) several attempts to quantize gravity have naturally led to the requirement of violations of the LI; 3) experiments on cosmic rays can represent the most easily approachable tool to probe the structure of space-time on the very small scales; 4) LI violations affect the thresholds for elementary processes relevant for cosmic ray astrophysics and can be observationally tested.

V. ACKNOWLEDGEMENTS

REFERENCES

[1] S. Coleman, S.L. Glashow, Phys. Rev. **D59**, 116008 (1999).

[2] J.A. Wheeler, Annals of Physics **2**, 604 (1957).

[3] A.I. Nikishov, Sov. Phys. - JETP **14**, 393 (1962).

[4] P. Goldreich and P. Morrison, Sov. Phys. - JETP **18**, 239 (1964).

[5] R.J. Gould and G.P. Schreder, Phys. Rev. Lett. **16**, 252 (1966).

[6] K. Greisen, Phys. Rev. Lett. **16**, 748 (1966).

[7] G.T. Zatsepin and V.A. Kuzmin, Pis'ma Zh. Ekps. Teor. Fiz. **4**, 114 (1966) [JETP Lett. **4**, 78 (1966)].

[8] D.A. Kirzhnits and V.A. Chechin, Sov. Jour. Nucl. Phys. **15**, 585 (1971).

[9] L. Gonzalez-Mestres, Proc. 26th ICRC (Salt Lake City, USA), **1**, 179 (1999).

[10] G.Yu. Bogoslovsky and H.F. Goenner, preprint gr-qc/9904081.

[11] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, preprint hep-th/9909085.

[12] J. Ellis, K. Farakos, N.E. Mavromatos, V.A. Mitsou and D.V. Nanopoulos, preprint astro-ph/9907340.

[13] F. Krennrich et al., Astrophys. J. **511**, 149 (1999).

[14] F.A. Aharonian et al., Astron. & Astroph. **349**, 11 (1999).

[15] V.V. Vassiliev, preprint astro-ph/9908088.

[16] F.W. Stecker, preprint astro-ph/9904416.

[17] P.S. Coppi and F.A. Aharonian, Astropart. Phys. **11**, 35 (1999).

[18] V.S. Berezinsky, P. Blasi and A. Vilenkin, Phys. Rev. **D58**, 103515 (1998).

[19] S.D. Biller et al., Phys. Rev. Lett. **83**, 2108 (1999).

[20] J. Lukierski, preprint hep-th/9812063.

[21] J. Lukierski, H. Ruegg, A. Nowicki and V.N. Tolstoy, Phys. Lett. **B264**, 331 (1998).

[22] J. Lukierski and H. Ruegg, Phys. Lett. **B329**, 189 (1994).

[23] P. Kosinski, J. Lukierski and P. Maslanka, preprint hep-th/9902037.

[24] W. Kluzniak, Astropart. Phys. **11**, 117 (1999).

[25] H. Rund, "The Differential Geometry on Finsler Spaces", Berlin 1959.

[26] M. Catanese et al., Astrophys. J. **469**, 572 (1996).

[27] P.L. Ghia et al., Nucl. Phys. B (Proc. Suppl.) **70**, 506 (1999).

[28] R.J. Protheroe and P.L. Biermann, Astropart. Phys. **6**, 45 (1996); Erratum-ibid **7**, 181 (1997).

[29] M. Takeda et al., Astrophys. J. **522**, 225 (1999).

[30] T. Abu-Zayyad et al., Proc. 26th ICRC (Salt Lake City, USA), **3**, 264 (1999).

[31] S.C. Corbato et al., Nucl. Phys. B (Proc. Suppl.) **28B**, 36 (1992).

[32] J.W. Cronin, Nucl. Phys. B (Proc. Suppl.) **28B**, 213 (1992).

	Infrared	Microwave	Radio
I_+	≈ 0.73	0.06	$5 \cdot 10^{-7}$
I_-	No solution	No solution	No solution
II_+	≈ 1	≈ 1	2×10^{-3}
II_-	≈ 1	≈ 1	No solution

TABLE I. Values of x that solve the equation for the threshold for pair production in the non-Lorentz invariant approach.

	Infrared	Microwave	Radio
I_+	$M \gtrsim 0.2 M_P$	$M \gtrsim 800 M_P$	$M \gtrsim 2.5 \times 10^{18} M_P$
I_-	$M \gtrsim 6 M_P$	$M \gtrsim 3 \times 10^4 M_P$	$M \gtrsim 8 \times 10^{19} M_P$
II_+	$(M \gtrsim 3 \times 10^{-8} M_P)$	$(M \gtrsim 7 \times 10^{-6} M_P)$	$M \gtrsim 10^5 M_P$
II_-	$(M \gtrsim 3 \times 10^{-7} M_P)$	$(M \gtrsim 10^{-4} M_P)$	$M \gtrsim 10^6 M_P$

TABLE II. Limits on the scale M where the LI is broken.

	x	Limit
I_+	2×10^{-5}	$M \gtrsim 3 \times 10^{13} M_P$
I_-	No solution	$M \gtrsim 10^{15} M_P$
II_+	0.02	$M \gtrsim 536 M_P$
II_-	No solution	$M \gtrsim 6 \times 10^3 M_P$

TABLE III. Values of x that solve the equations for the threshold of photopion production on the microwave background (first column) and lower limit on the scale M of breaking of LI (second column).